1 Fig. 8 shows the line $y = 1$ and the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \frac{(x-2)^2}{x}$. The curve touches the x-axis at P(2, 0) and has another turning point at the point Q.

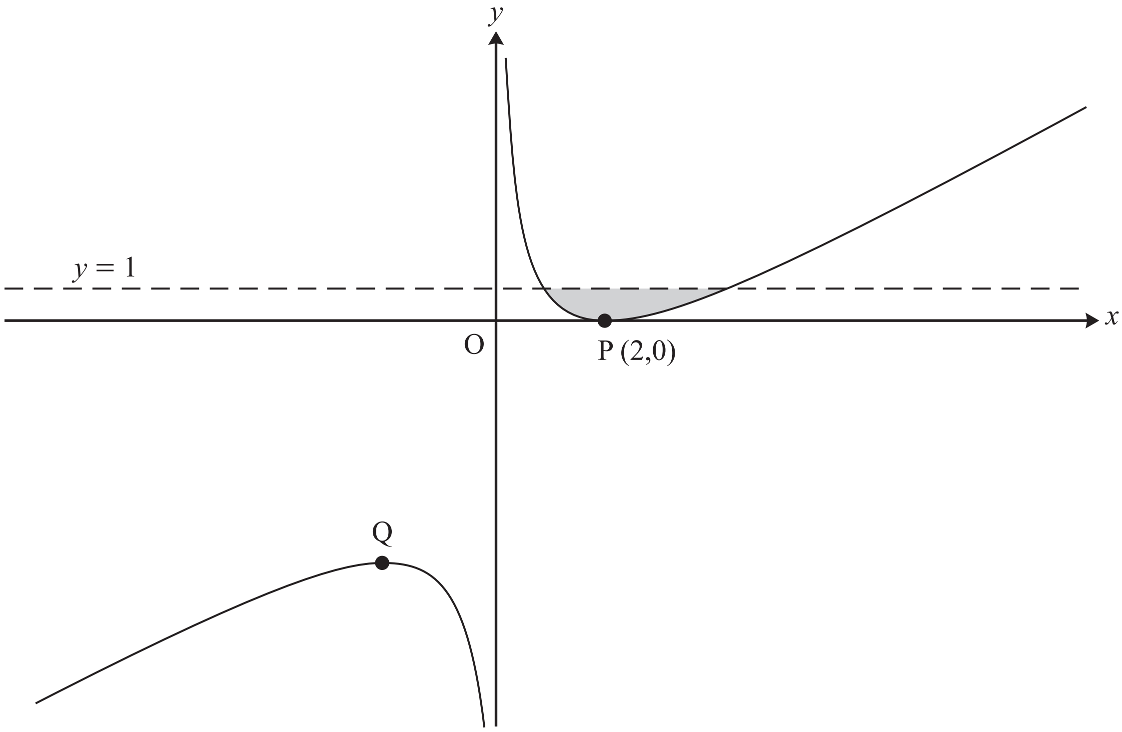

Fig. 8

(i) Show that $\mathrm{f}'(x) = 1 - \frac{4}{x^2}$, and find $\mathrm{f}''(x)$.

Hence find the coordinates of Q and, using $\mathrm{f}''(x)$, verify that it is a maximum point. **[7]**

(ii) Verify that the line $y = 1$ meets the curve $y = \mathrm{f}(x)$ at the points with x-coordinates 1 and 4. Hence find the exact area of the shaded region enclosed by the line and the curve. **[6]**

The curve $y = \mathrm{f}(x)$ is now transformed by a translation with vector $\begin{pmatrix} -1 \\ -1 \end{pmatrix}$. The resulting curve has equation $y = \mathrm{g}(x)$.

(iii) Show that $\mathrm{g}(x) = \frac{x^2 - 3x}{x+1}$. **[3]**

(iv) Without further calculation, write down the value of $\int_0^3 \mathrm{g}(x)\,\mathrm{d}x$, justifying your answer. **[2]**

2 Fig. 9 shows the curve $y = xe^{-2x}$ together with the straight line $y = mx$, where m is a constant, with $0 < m < 1$. The curve and the line meet at O and P. The dashed line is the tangent at P.

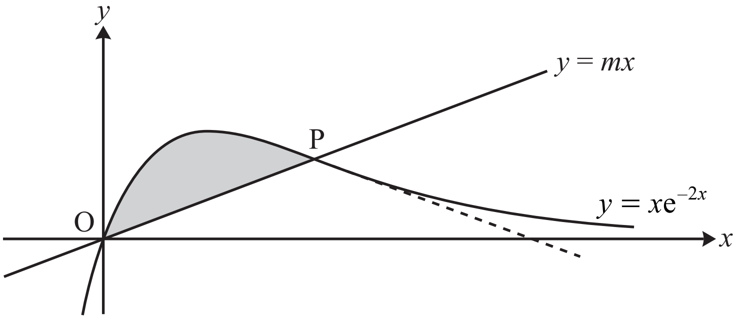

Fig. 9

(i) Show that the x-coordinate of P is $-\frac{1}{2}\ln m$. **[3]**

(ii) Find, in terms of m, the gradient of the tangent to the curve at P. **[4]**

You are given that OP and this tangent are equally inclined to the x-axis.

(iii) Show that $m = e^{-2}$, and find the exact coordinates of P. **[4]**

(iv) Find the exact area of the shaded region between the line OP and the curve. **[7]**

END OF QUESTION PAPER

3 **(i)** Use the substitution $u = 1 + x$ to show that

$$\int_0^1 \frac{x^3}{1+x}\,dx = \int_a^b \left(u^2 - 3u + 3 - \frac{1}{u}\right) du,$$

where a and b are to be found.

Hence evaluate $\displaystyle\int_0^1 \frac{x^3}{1+x}\,dx$, giving your answer in exact form. **[7]**

Fig. 8 shows the curve $y = x^2 \ln(1 + x)$.

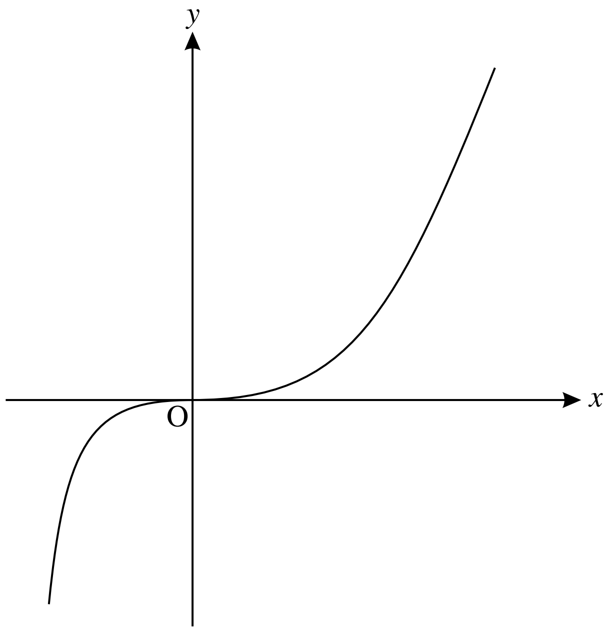

Fig. 8

(ii) Find $\dfrac{dy}{dx}$.

Verify that the origin is a stationary point of the curve. **[5]**

(iii) Using integration by parts, and the result of part **(i)**, find the exact area enclosed by the curve $y = x^2 \ln(1 + x)$, the x-axis and the line $x = 1$. **[6]**